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Preface

This final scientific report summarizes a two-years investigation effort (1.9.1986-31.8.1988) sponsored by AFOSR Grant No. 86-0355. In order to enhance the usefulness of the report for individuals of different interests and responsibilities, it is divided into three parts.

The first part (I) outlines in generic terms the research concept which has lead to the development of an innovative approach for missile guidance law synthesis; it summarizes the main results achieved by the two-year effort and indicates directions deserving further investigation.

The second part (II), which can be used as an independent scientific document, concentrates on the theoretical aspects of the problem formulation and outlines the mathematical framework for the mixed guidance law synthesis.

The third part (III) is fully application oriented. It describes in detail the model used for the investigation and the process of interactive guidance law synthesis. Explicit guidelines for a potential user are given. Several examples, demonstrating the performance improvement which can be achieved by using the proposed approach, are also included in this part.

Abstract

The terminal phase of the encounter between a radar guided missile and a highly maneuverable aircraft, which can employ also electronic countermeasures, is formulated as an imperfect information zero-sum pursuit-evasion game played between the missile designer and the pilot of the target aircraft. For this scenario a new method of guidance law synthesis, based on the concept of optimal mixed strategies, is developed and implemented. The "mixed" guidance law design approach is based on a rigorous mathematical framework and leads to feasible solutions which guarantee that the single shot kill probability (SSKP) of the missile is higher than the value achievable by any other presently used guidance law.

STOCHASTIC GAME APPROACH TO GUIDANCE DESIGN

PART I: CONCEPT OUTLINE AND SUMMARY

1.1 Introduction

The subject of the reported investigation is the terminal guidance problem of a medium-range radar guided missile (air-to-air or surface-to-air) fired against a highly maneuverable aircraft, which may also use electronic countermeasures to enhance its survivability. In the last two decades several works, dealing with different aspects of the terminal phase of such an encounter, were published. Some papers formulated this engagement as a one-sided guidance [1-8] or avoidance [9-13] problem.

Classical guidance laws (such a proportional navigation or three-point guidance) have been shown to be susceptible to optimal target maneuvers. Deterministic missile avoidance strategies [9-11] and even periodical random phase maneuvers of an optimal frequency [12] have the potential to create miss distances larger than the lethal range of the missile warhead. In modern guidance laws, based on optimal control theory [1-3, 5], target maneuver is assumed to be observable or known and it is included in the calculation of the missile's zero-effort miss. The performance of such guidance laws are strongly dependent on the quality of target maneuver estimation [4, 7, 8]. In fact, the missile versus aircraft engagement is a typical pursuit-evasion game and should be addressed as such. If perfect information can be assumed, it is a deterministic zero-sum differential game [14, 15]; otherwise it has to be formulated in a stochastic setting [16-23].

The solution of the perfect information game indicates that a well designed missile, having an appropriate maneuverability advantage, can attain negligibly small miss distances against any feasible target maneuver. The logical conclusion drawn from this

result is that aircraft survivability can be enhanced only by information deterioration or ultimately by information denial. Fortunately for the aircraft, the measurements taken by a radar guided missile are always corrupted by noise. The designer of such a missile has to incorporate an estimator (or filter) in the guidance loop. The estimator design needs assumptions on target maneuvering. The better these assumptions match the actual maneuver, the better is the missile performance. This observation leads to conclude that survival enhancement requires maximization of the estimation error of the missile. Both an unpredictable random maneuvering of the aircraft and an artificially generated random target motion by ECM [13] can serve this purpose of reducing the a priori information available to the estimator.

In summary one can say, using terminology of game theory, that the optimal strategy of the target aircraft for survivability enhancement against a well designed radar guided missile is a random or "mixed strategy" [24, 25]. Similar conclusions were reached, at least partially, in some previous studies [27-29].

From a game theoretical point of view, if the optimal strategy of one player is mixed, the entire game solution becomes probabilistic and in general the other player's optimal strategy is also mixed. However, many previous works [18-23] investigated imperfect information games searching for solutions in pure strategies. Moreover, no attempt is known to consider a mixed strategy for the missile in the terminal phase of a homing encounter.

1.2 Research Objectives

In order to obtain optimal missile guidance strategies which are "secure" [22, 24] (i.e. guarantee the highest level of success in a stochastic sense) and have a feasible implementation, an innovative concept was outlined in the original research proposal submitted to AFOSR on Oct. 3, 1985. This new approach allows the missile designer to use mixed guidance strategies for maximizing the "single shot kill probability" (SSKP) of the missile. For the two-years investigation the following objectives were set:

a.   Identify and prove, based on a rigorous mathematical analysis, the properties of the optimal mixed strategy elements, i.e. the optimal pure strategy set, needed for guidance law synthesis.

b.   Outline a feasible design procedure for mixed guidance law synthesis.

c.   Demonstrate by a set of examples the performance improvement achieved by the mixed strategy approach in comparison with other "modern" guidance laws.

1.3 Methodology

The first step of the investigation was to formulate the terminal phase of the homing encounter as an imperfect information zero-sum pursuit-evasion game between the missile designer (pursuer) and the pilot of the target aircraft (evader), where both are allowed to use a mixed strategy. As the pay-off function of the game the single shot kill probability (SSKP) of the missile was selected. It is to be maximized by the pursuer and minimized by the evader. Note that this pay-off function, which is indeed the one of genuine interest in air warfare (rather than the

mathematically convenient linear quadratic expressions), has not been used in previous works.

The admissible controls of the evader in this game are the lateral acceleration of the aircraft, constrained only by structural or aerodynamical limits, and the use of ECM in the form of "electronic jinking". For the sake of simplicity it is assumed that the elements of the evader's "pure strategy set" (i.e. maneuver and ECM options) are countable.

In this formulation the problem faced by the missile designer is to find the "optimal strategy set" of the pursuer, i.e. the number and the type of guidance laws to be programmed into the missile for achieving optimal performance. The "optimal mixed strategies", representing the solution of the above formulated game, are probability distributions on the respective pure strategy sets [24].

The "rules of the game" are such that at the outset of each encounter respective chance mechanisms, which implement the optimal mixed strategies, select an element of each pure strategy set to be used during the entire engagement. Due to the short duration of the end-game encounter, these rules provide the only reasonable way for acting in a future operational scenario.

The countability of the pure strategy sets and the "rules of the game" transform the originally very complex stochastic differential game to a "matrix game", which is solvable by well known linear programming methods [31].

1.4 Summary of Results

1.4.1 First phase (1.9.1986-31.5.87)

In the first phase of the investigation under the AFOSR Grant
the efforts were focused on creating a rigorous mathematical
framework for the mixed missile guidance synthesis.

After demonstrating by some simple examples that in certain
circumstances a mixed strategy performs better than any of the
available pure strategies [32-33], a game theoretical study was
initiated.

This mathematical investigation was based on introducing the
notion of "k-optimal" strategy sets (optimal strategy sets
composed of k active elements). The basic properties that such
sets (if they exist) have to satisfy were derived and proven. In
the sequel a theorem was formulated to demonstrate the convergence
of these sets with increasing $k \geq 1$ to the optimal strategy set
(which can be infinite, though countable). This theorem implies
that even an infinite optimal strategy set can be approximated by
a finite "k-optimal" set.

By proving an additional theorem, a constructive iterative
procedure for finding the finite approximation of the optimal
strategy set was developed. These results were presented as an
invited paper at the 25th CDC in Athens [34]. A refined version
of this paper was submitted for an archive journal publication
[35]. Details included in these papers are presented in Part II
of this report.

Since the properties of the "k-optimal" strategy sets,
derived by the mathematical analysis, cannot be obtained by
guidance laws using estimators of modest dimensions, they actually

serve as guidelines for a feasible design.

The first example based on such guidelines worked out for an ECM-free planar scenario yielded a pure optimal guidance strategy (it is a particular case with k=1). Nevertheless this guidance law performed better i.e. guaranteed a higher SSKP than any other previously used one, because it did not have any weak spot.. Such an example was presented at the Second International Symposium on Differential Game Applications [36]. However, some other examples which included "electronic jinking", indicated that in an ECM environment a mixed guidance strategy is probably the onl·/ one to provide acceptable missile performance.

1.4.2 Second phase (1.5.-31.10.1987)

Based on the results of the last examples the research activity during the subsequent period was focused on investigating the interaction of kinematic target maneuver and electronic jinking. Results of several planar engagements were presented at the AIAA Guidance and Control Conference in Monterey [37]. The paper based on this presentation [38] was accepted and scheduled for publication in the Journal of Guidance, Control and Dynamics for early 1989.

Interpretation of the planar results raised the questions on the validity of such a model. Since lateral accelerations and electronic jinking take place in planes more or less perpendicular to each other, a three-dimensional analysis seemed to be necessary.

1.4.3 Third phase (1.11.1987-31.8.1988)

As a consequence of the concern which emerged from the interpretation of the planar engagement results, the major effort

during the final phase of the investigation was concentrated on verifying the mixed strategy guidance concept in a three-dimensional scenario with eventual use of electronic jinking. The results clearly indicated that a mixed guidance law optimized for planar geometry does not perform well in a three-dimensional case. For an acceptable performance the mixed guidance law synthesis has to be carried out using a three-dimensional engagement model. Examples of the three-dimensional design concept were presented in an invited paper at ACC-88 in Atlanta [39]. Further results emphasizing the need for a three-dimensional mixed guidance law synthesis are included in Part III of this report.

1.5 Topics for Further Research

Due to the limited time and resources allocated for the project, many interesting topics evolving from the otherwise successful research could not be investigated. There is no doubt that for an efficient implementation of the mixed guidance strategy concept in a real weapon system a more profound and detailed study is needed.

The most important one is probably a parametric sensitivity analysis with a given mixed strategy structure. The major parameters in such a study seem to be the following:

a) warhead lethality radius,

b) missile/target maneuver ratio,

c) nonlinear maneuver similarity parameter ($a_{E_{max}} \tau_P^2$),

d) normalized end-game duration,

e) initial end-game geometry.

Other topics, which may deserve attention, are:

(i) effects of variable missile speed,

(ii) effects of missile dynamics (order of transfer function,
      non-minimal phase, etc.),

(iii) influence of estimator structure and dimension with fixed
      gains,

(iv) the effect of time varying Kalman gains.

This is by no means a complete list of research topics
deserving investigation, but further details in this direction
seem to be out of the scope of the present report.

1.6 Conclusions

At the completion of the intensive two-years research effort
it can be stated that the investigation has achieved its highly
set objectives:

a)   A solid mathematical framework for the mixed guidance
     strategy concept has been established.

b)   A practical design procedure for mixed guidance law synthesis
     has been outlined and implemented in a skid-to-turn missile of
     first-order dynamics.

c)   The robust performance of a mixed guidance strategy, superior
     to any known missile guidance law, has been demonstrated.

The original contributions of the reported research are the
new formulation of the missile-aircraft end-game and the creation
of a revolutionary, but nevertheless feasible, guidance law
synthesis.

Results of the above activity are summarized in five
conference papers [32, 34, 36, 37, 39] and in three archive
journal publications [33, 35, 38]. The investigation also created
an enhanced insight in terminal guidance problems and identified a
list of new topics deserving further study.

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